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A FORM OF AERODYNAMIC ADMITTANCE FOR USE IN BRIDGE AEROELASTIC ANALYSIS

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This paper returns to, and addresses, the question of identifying the nature of *aerodynamic admittance* in relation to extended-span bridges in wind. Theoretical formulations for the sectional aerodynamic forces acting upon the deck girder of a long-span bridge have conventionally been composed of the sum of two kinds of terms: aeroelastic terms and buffeting terms. The former employ frequency-dependent coefficients ("flutter derivatives") associated with sinusoidal displacements of the structure, while the latter have typically been expressed in quasi-static terms with fixed lift, drag and moment coefficients. This inconsistency of formulation has required that at some point the buffeting terms, functions of gust velocity, be adjusted to a more compatible form through the introduction of the so-called aerodynamic admittance factors that are frequency-dependent. The present paper identifies a form of these several section-force factors as functions of the flutter derivatives themselves.

1. INTRODUCTION

At the present time extensive theory exists for forecasting the aeroelastic response of long-span bridges. Data in support of this theory are available in three categories:

(i) static and motional force coefficients measured on a wind-tunnel section model of the deck girder; (ii) computed natural bridge modes and frequencies of the whole structure; and (iii) spectral and coherence descriptions of the cross-wind.

As has been repeatedly pointed out in the literature (Scanlan & Tomko 1971; Scanlan, Béliveau & Budlong 1974; Scanlan, Jones & Singh 1997; Simiu & Scanlan 1996), transferring the application of theoretical thin-airfoil motional force coefficients to the bluff bodies presented by bridge deck sections is *basically incorrect* from both mathematical and physical viewpoints. Thin airfoil dynamic force theories depend on unique *circulation functions* [such as those of Theodorsen (1935) and Sears (1941)] that cannot be realized for bluff bodies that experience separated flow.

It is therefore taken here for granted that the common use of the Sears admittance function in the context of bridge deck buffeting can, at best, be viewed only as a suggested approximation. The Sears admittance function represents the dimensionless theoretical frequency-dependent force spectral level of a thin airfoil, with wholly attached flow, penetrating a vertically oscillating gust field. While the use of the Sears function by Liepmann (1952) in the airfoil context is technically correct, that by Davenport (1962) in the bridge context is not. The present paper first briefly reviews the main elements in the description of the aerodynamic forces, emphasizing in particular the relation between a form of aerodynamic admittance and the flutter derivatives. How these relations enter the consequent aeroelastic analysis is then suggested.

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2. CONVENTIONAL SECTIONAL AERODYNAMIC THEORY

In the case of a bridge deck section with degrees of freedom h (vertical), p (lateral), and α (twist) it has become conventional to separate the associated time-dependent sectional force vectors into aeroelastic (ae) and buffeting (b) contributions. Thus,

$$\text{vertical lift:} \quad L = L_{ae} + L_b, \tag{1}$$

$$\text{horizontal drag:} \quad D = D_{ae} + D_b, \tag{2}$$

$$\text{moment:} \quad M = M_{ae} + M_b, \tag{3}$$

where, for sinusoidal motion, the aeroelastic terms involve, in commonly used form (Sarkar *et al.* 1994), up to six frequency-dependent aeroelastic or flutter derivative terms H_i^*, P_i^*, A_i^* ($i = 1, \ldots, 6$) within each force component:

$$L_{ae} = \frac{1}{2}\rho U^2 B\left[KH_1^*\frac{\dot{h}}{U} + KH_2^*\frac{B\dot{\alpha}}{U} + K^2H_3^*\alpha + K^2H_4^*\frac{h}{B} + KH_5^*\frac{\dot{p}}{U} + K^2H_6^*\frac{p}{B}\right], \tag{4}$$

$$D_{ae} = \frac{1}{2}\rho U^2 B\left[KP_1^*\frac{\dot{p}}{U} + KP_2^*\frac{B\dot{\alpha}}{U} + K^2P_3^*\alpha + K^2P_4^*\frac{p}{B} + KP_5^*\frac{\dot{h}}{U} + K^2P_6^*\frac{h}{B}\right], \tag{5}$$

$$M_{ae} = \frac{1}{2}\rho U^2 B^2\left[KA_1^*\frac{\dot{h}}{U} + KA_2^*\frac{B\dot{\alpha}}{U} + K^2A_3^*\alpha + K^2A_4^*\frac{h}{B} + KA_5^*\frac{\dot{p}}{U} + K^2A_6^*\frac{p}{B}\right] \tag{6}$$

in which ρ is the air density, U the mean cross-wind velocity, B the deck width, $K = B\omega/U$, and ω is the circular frequency of oscillation. This large array provides in advance for all possible motional contributions. Experimental evaluation brings out which of the flutter derivatives are important and which are not. Hence some formulations have been limited to fewer terms.

The buffeting forces have been written conventionally (Scanlan & Jones 1990) in quasi-static terms:

$$\text{vertical:} \quad L_b = \frac{1}{2}\rho U^2 B\left[2C_{L_0}\frac{u}{U} + C'_{L_0}\frac{w}{U}\right], \tag{7}$$

$$\text{horizontal:} \quad D_b = \frac{1}{2}\rho U^2 B\left[2C_{D_0}\frac{u}{U} + C'_{D_0}\frac{w}{U}\right], \tag{8}$$

$$\text{moment:} \quad M_b = \frac{1}{2}\rho U^2 B^2\left[2C_{M_0}\frac{u}{U} + C'_{M_0}\frac{w}{U}\right], \tag{9}$$

where C_L, C_D, C_M represent force coefficients, C'_L, C'_D, C'_M their slopes versus wind vertical attack angle α, and u and w the along-wind and vertical gust velocity components, respectively. In this formulation C_L and C_D are in perpendicular and horizontal directions, respectively. This conventional writing of the buffeting forces implies association of a unit aerodynamic admittance factor with each force component. This simplified, useful, but often incomplete formulation will be modified at a later point.

3. HEURISTIC REEXAMINATION OF SECTIONAL FORCES

The basic conceptual formula for sectional lift at any instant is

$$L = \frac{1}{2}\rho U_r^2(t) BC_\mathrm{L}(t), \tag{10}$$

where U_r and C_L may be considered variable, U_r representing the relative horizontal wind velocity, lift L being understood as *vertical*.

If the structural section has displacement components h vertical and p horizontal, and the gust velocity components are u and w, then one may write

$$U_r = U + u - \dot{p}, \tag{11}$$

while C_L may be represented as a "base" value C_{L_0} plus an increment due to a small angular change $w - \dot{h}/U$:

$$C_L(t) = C_{L_0} + C'_{L_0}\left(\frac{w - \dot{h}}{U}\right) \tag{12}$$

for $C'_{L_0} = \mathrm{d}C_{L_0}/\mathrm{d}\alpha$, where C_{L_0} accounts for all lift contributions—such as mean, steady-state or signature (internal structure-induced) effects—not due to the explicit horizontal and vertical causes mentioned. Thus,

$$L = \frac{1}{2}\rho B[U + u - \dot{p}]^2\left[C_{L_0} + C'_{L_0}\left(\frac{w - \dot{h}}{U}\right)\right], \tag{13}$$

which, if products of small quantities relative to U are neglected, becomes

$$L = \frac{1}{2}\rho U^2 B\left[C_{L_0} + C'_{L_0}\left(\frac{w - \dot{h}}{U}\right) + 2C_{L_0}\left(\frac{u - \dot{p}}{U}\right)\right]. \tag{14}$$

This then represents a base value plus the variable lift contributions ascribable to the specific horizontal and vertical effects mentioned.

It is important to emphasize the concept of gusting that is inherent in the above formulation. Both the horizontal and vertical velocities u, w are assumed in this writing to extend unchanged spatially over the full height or width, respectively, of the bridge deck section in question. In other words, the assumption implicit in these expressions is that, over the extent of the deck section geometry in either direction, vertical or horizontal, the gust velocity is completely spatially coherent. This is a conservative assumption regarding developed force values. It could, if desired, be modified through the introduction of coherence postulates, which will not be done here.

In equations (4)–(6) the assumed motion of h, p and α is purely sinusoidal. If this is represented in the form $\mathrm{e}^{\mathrm{i}\omega t}$ then the particular freedoms h and p may be represented in terms of $\dot{h}$ and $\dot{p}$ as $\dot{h}/\mathrm{i}\omega$, $\dot{p}/\mathrm{i}\omega$, respectively.

The time-varying terms of equation (14) may now be compared to the u, w, $\dot{h}$ and $\dot{p}$ terms in equations (1), (4) and (7), according to which the terms associated with vertical and horizontal contributions to lift are, together:

$$L_{hp} = \frac{1}{2}\rho U^2 B\left[K[H_1^* - \mathrm{i}H_4^*]\frac{\dot{h}}{U} + K[H_5^* - \mathrm{i}H_6^*]\frac{\dot{p}}{U} + 2C_{L_0}\frac{u}{U} + C'_{L_0}\frac{w}{U}\right]. \tag{15}$$

This suggests that in equation (14) the following equivalences are appropriate:

$$2C_{L_0} = -K[H_5^* - \mathrm{i}H_6^*], \tag{16}$$

$$C'_{L_0} = -K[H_1^* - \mathrm{i}H_4^*], \tag{17}$$

These recognize the inherent phasing and frequency dependence required of C_{L_0}, C'_{L_0}.

By analogous reasoning applied to drag and moment expressions, the following equivalences are also suggested:

$$2C_{D_0} = -K[P_1^* - \mathrm{i}P_4^*], \tag{18}$$

$$C'_{D_0} = -K[P_5^* - \mathrm{i}P_6^*], \tag{19}$$

$$2C_{M_0} = -K[A_5^* - \mathrm{i}A_6^*], \tag{20}$$

$$C'_{M_0} = -K[A_1^* - \mathrm{i}A_4^*], \tag{21}$$

This conversion alters the buffeting force terms to the form

$$\frac{L_b}{\frac{1}{2}\rho U^2 B} = -K[H_5^* - \mathrm{i}H_6^*]\frac{u}{U} - K[H_1^* - \mathrm{i}H_4^*]\frac{w}{U}, \tag{22}$$

$$\frac{D_b}{\frac{1}{2}\rho U^2 B} = -K[P_1^* - \mathrm{i}P_4^*]\frac{u}{U} - K[P_5^* - \mathrm{i}P_6^*]\frac{w}{U}, \tag{23}$$

$$\frac{M_b}{\frac{1}{2}\rho U^2 B^2} = -K[A_5^* - \mathrm{i}A_6^*]\frac{u}{U} - K[A_1^* - \mathrm{i}A_4^*]\frac{w}{U} \tag{24}$$

From these, the autospectrum S_{LL} of lift becomes

$$\frac{S_{LL}}{[\frac{1}{2}\rho UB]^2} = K^2[H_5^{*2} + H_6^{*2}]S_{uu} + K^2[H_5^* - \mathrm{i}H_6^*][H_1^* + \mathrm{i}H_4^*]S_{uw} + K^2[H_5^* + \mathrm{i}H_6^*][H_1^* - \mathrm{i}H_4^*]S_{wu} + K^2[H_1^2 + H_4^2]S_{ww}, \tag{25}$$

where S_{uw} is the cross-spectrum of u and w.

Those for drag and moment are constituted similarly:

$$\frac{S_{DD}}{[\frac{1}{2}\rho UB]^2} = K^2[P_1^{*2} + P_4^{*2}]S_{uu} + K^2[P_5^* - \mathrm{i}P_6^*][P_1^* + \mathrm{i}P_4^*]S_{uw} + K^2[P_5^* + \mathrm{i}P_6^*][P_1^* - \mathrm{i}P_4^*]S_{wu} + K^2[P_5^{*2} + P_6^{*2}]S_{ww}, \tag{26}$$

$$\frac{S_{MM}}{[\frac{1}{2}\rho UB^2]^2} = K^2[A_5^{*2} + A_6^{*2}]S_{uu} + K^2[A_5^* - \mathrm{i}A_6^*][A_1^* + \mathrm{i}A_4^*]S_{uw} + K^2[A_5^* + \mathrm{i}A_6^*][A_1^* - \mathrm{i}A_4^*]S_{wu} + K^2[A_1^{*2} + A_4^{*2}]S_{ww}, \tag{27}$$

where S_{uw} is the cross spectrum of u and w.

If earlier formulations (22)–(24) are referred to, and the cross-spectra of u and w are neglected, the results corresponding to expressions (25–27) above were

$$\text{lift:}\quad \frac{S_{LL}}{[\frac{1}{2}\rho UB]^2} = 4C_{L_0}^2\chi_L^2 S_{uu} + (C'_L)^2\chi_{L'}^2 S_{ww}, \tag{28}$$

$$\text{drag:}\quad \frac{S_{DD}}{[\frac{1}{2}\rho UB]^2} = 4C_{D_0}^2\chi_D^2 S_{uu} + (C'_{D_0})^2\chi_{D'}^2 S_{ww}, \tag{29}$$

$$\text{moment:}\quad \frac{S_{MM}}{[\frac{1}{2}\rho UB^2]^2} = 4C_{M_0}^2\chi_M^2 S_{uu} + (C'_{M_0})^2\chi_{M'}^2 S_{ww}, \tag{30}$$

where frequency-dependent correction factors χ^2, *aerodynamic admittances*, have been incorporated to account for frequency dependence. This formulation is seen to be superseded by equations (25)–(27). In fact the notion of aerodynamic admittance is clarified by this process; it suggests that *a concept of aerodynamic admittance can be viewed as defined in terms of flutter derivatives*, which imply the following definitions:

$$4C_L^2\chi_L^2 = K^2[H_5^{*2} + H_6^{*2}], \tag{31}$$

$$4C_D^2\chi_D^2 = K^2[P_1^{*2} + P_4^{*2}], \tag{32}$$

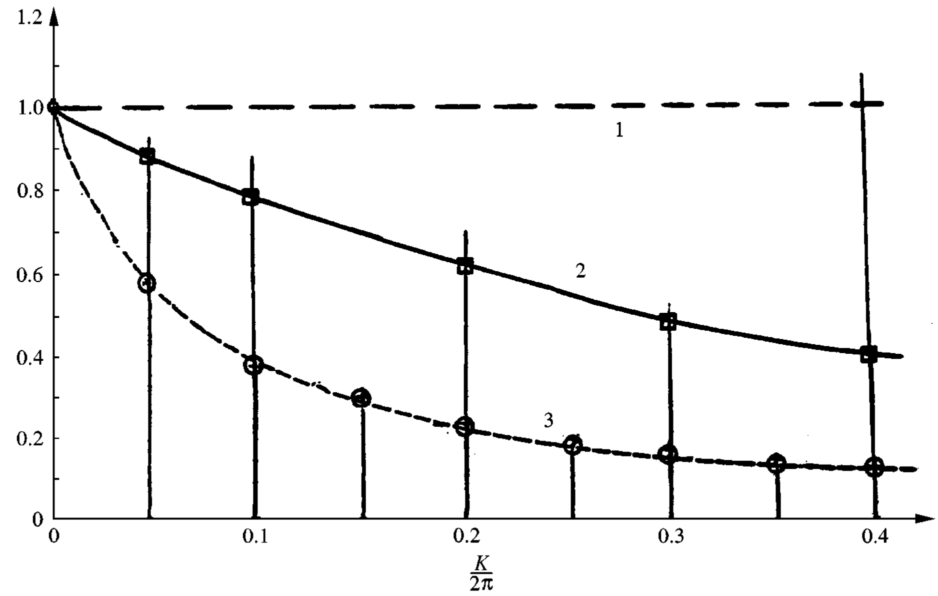

Figure 1. Representative aerodynamic admittances: (1) unitary (for quasi-static gust forces); (2) example corresponding to H_1^* of Figure 2; (3) classic Sears function.

$$4C_M^2\chi_M^2 = K^2[A_5^{*2} + A_6^*], \tag{33}$$

$$(C_L')^2\chi_{L'}^2 = K^2[H_1^{*2} + H_4^{*2}], \tag{34}$$

$$(C_D')^2\chi_{D'}^2 = K^2[P_5^{*2} + P_6^{*2}], \tag{35}$$

$$(C_M')^2\chi_{M'}^2 = K^2[A_1^{*2} + A_4^{*2}], \tag{36}$$

These formulas suggest relationships that have not to-date been widely demonstrated by available data. Two brief illustrations may, however, be given. If H_1^* is proportional to $2\pi/K$ (as is sometimes the case) and H_4^* is negligible (also reasonable), then the admittance $\chi_{L'}^2$ has a constant unit value as in quasi-steady theory; (see curve 1, Figure 1). If, as a further example, H_1^* is as illustrated in Figure 2 for a realistic case for which $C_L' = 4{\cdot}11$ per radian, with H_4^* again negligible then $\chi_{L'}^2$ evolves as shown in curve 2, Figure 1, which evidences a trend resembling somewhat that of the Sears function, curve 3 of Figure 1.

The point of the present discussion is to open up a promising vector of investigation.

4. TIME-DEPENDENT ANALYSIS OF SECTIONAL AERODYNAMIC EFFORTS

The discussion in this section essentially reviews and arrives at the same results as the analysis of the preceding section but from a *transient*-aerodynamic viewpoint. *Lift* as a representative case will be examined first.

Consider a deck section undergoing a vertical velocity $\dot{h}(t)$ and simultaneously the impact of a vertical gust $w(t)$. The net effective vertical wind angle of attack θ is then

$$\theta = \frac{w - \dot{h}}{U} \tag{37}$$

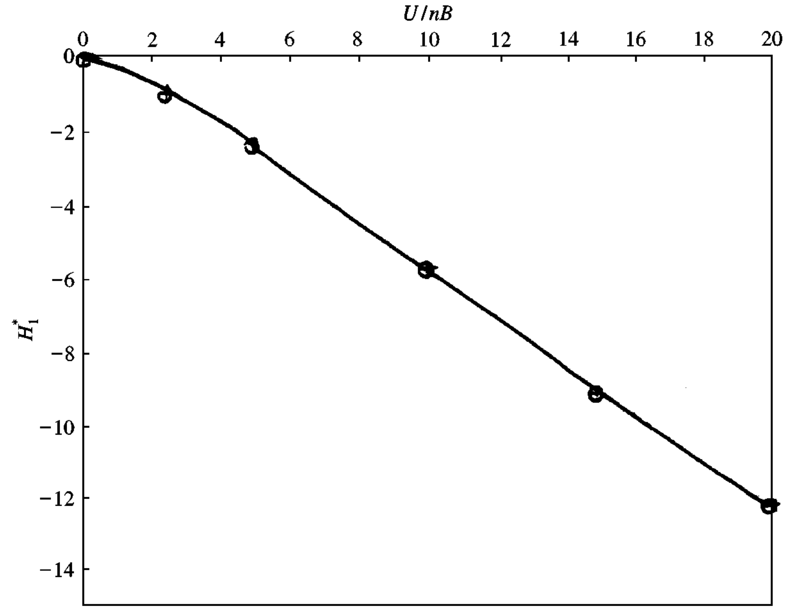

Figure 2. Typical flutter derivative H_1^* for bridge deck.

and the corresponding transient lift is

$$L(s) = \tfrac{1}{2}\,\rho U^2 B C'_L I(s), \tag{38}$$

where $s = Ut/B$ is dimensionless time, $C'_L = \mathrm{d}C_L/\mathrm{d}\theta$ and $I(s)$ is the lift-growth integral:

$$I(s) = \int_{-\infty}^{s} \Phi(s-\sigma)\,\theta'(\sigma)\,\mathrm{d}\sigma \tag{39}$$

in which $\theta'(\sigma) = \mathrm{d}\theta/\mathrm{d}\sigma$ and $\Phi(s)$ is an appropriate "indicial" function.

4.1. The Flutter Case, $w = 0$

Information on Φ may be obtained from the standard flutter case in which $w = 0$, conventionally written (4) as

$$L = \frac{1}{2}\,\rho U^2 B\left[KH_1^* - \mathrm{i}KH_4^*\right]\frac{\dot{h}}{U}, \tag{40}$$

where H_1^* is a flutter derivative and $\dot{h}$ is purely sinusoidal.

With the change of variable $s - \sigma = \tau$, $I(s)$ may be written as

$$I(s) = \int_0^{\infty} \Phi(\tau)\,\theta'(s-\tau)\,\mathrm{d}\tau = \int_0^{\infty} \theta(s-\tau)\,\Phi'(\tau)\,\mathrm{d}\tau \tag{41}$$

in which $\Phi'(\tau) = \mathrm{d}\Phi/\mathrm{d}\tau$ so that $\theta = \theta_0 \mathrm{e}^{\mathrm{i}Ks}$:

$$L(s) = \tfrac{1}{2}\,\rho U^2 B C'_L\left[\Phi(0) + \overline{\Phi'}\right]\theta_0 \mathrm{e}^{iks} = \tfrac{1}{2}\,\rho U^2 B\left[KH_1^* - \mathrm{i}KH_4^*\right]\theta_0 \mathrm{e}^{\mathrm{i}Ks}, \tag{42}$$

where

$$\overline{\Phi'} = \int_0^\infty \Phi'(\tau)\,\mathrm{e}^{-\mathrm{i}K\tau}\,\mathrm{d}\tau \tag{43}$$

is the Fourier transform of $\Phi'(\tau) = \mathrm{d}\Phi/\mathrm{d}\tau$.

This yields the net relation

$$C'_L[\Phi(0) + \overline{\Phi'}] = K[H_1^* - \mathrm{i}H_4^*]. \tag{44}$$

4.2. The Buffeting Case, $\dot{h} = 0$

We now consider the alternate case in which $\theta = w/U$ and $\dot{h} = 0$.

In this case

$$L(s) = \frac{1}{2}\rho U^2 B C'_L\left[\Phi(0)\frac{w(s)}{U} + \int_0^\infty \frac{w(s-\tau)}{U}\Phi'(\tau)\,\mathrm{d}\tau\right], \tag{45}$$

of which the Fourier transform is

$$\bar{L} = \frac{1}{2}\rho U^2 B C'_L[\Phi(0) + \overline{\Phi'}]\frac{\bar{w}}{U} = \frac{1}{2}\rho U^2 B\frac{\bar{w}}{U}[KH_1^* - \mathrm{i}KH_4^*], \tag{46}$$

Multiplying $\bar{L}$ by its complex conjugate, it can immediately be demonstrated that the auto-PSD of lift is

$$S_{LL} = (\tfrac{1}{2}\rho U B)^2 K^2[H_1^2 + H_4^2]\,S_{ww}. \tag{47}$$

Recalling the quasi-steady expression for buffeting lift,

$$\frac{L}{\frac{1}{2}\rho U^2 B} = C'_L\frac{w}{U} \tag{48}$$

which leads to the lift spectrum

$$\frac{S_{LL}}{(\frac{1}{2}\rho U B)^2} = (C'_L)^2 S_{ww}\chi_{L'}^2 \tag{49}$$

as "corrected" by the admittance factor $\chi_{L'}^2$, it can be seen that the latter is defined [see equation (25)] by

$$\chi_{L'}^2 = K^2[H_1^2 + H_4^2]/(C'_\mathrm{L})^2, \tag{50}$$

where the relation of $\chi_{L'}^2$ to Φ' is

$$\chi_{L'}^2 = [\Phi(0) + \overline{\Phi'}]\,[\Phi(0) + \overline{\Phi'^*}]. \tag{51}$$

Clearly, the analogous expressions for admittances χ^2 in equations (31)–(36) are derivable by similar methods. The analysis of this section, while confirming that of the preceding, further links aerodynamic admittance directly to its associated indicial function source. Connections of this character were underscored by Scanlan (1984, 1993).

5. SINGLE-MODE FLUTTER AND BUFFETING

Although multi-mode analyses are now routinely performed (Jain *et al.* 1996; Katsuchi *et al.* 1998a, b), attention will be confined here to some remarks on the general process, with focus only on single-mode response.

5.1. Equation of Motion

This, with arbitrarily limited choice of flutter derivatives, is

$$I_i[\ddot{\xi}_i + 2\zeta_i\omega_i\dot{\xi}_i + \omega_i^2\xi_i] = Q_i, \tag{52}$$

where I_i is the whole-bridge generalized inertia of mode i, and,

$$Q_i = \frac{1}{2}\rho U^2 B^2 \ell \left\{ \frac{KB}{U}[H_1^* G_{h_i h_i} + P_1^* G_{p_i p_i} + A_2^* G_{\alpha_i \alpha_i}]\dot{\xi}_i. \right.$$

$$\left. + K^2 A_3^* G_{\alpha_i\alpha_i}\xi_i + \int_{\text{deck}} [\mathscr{L}h_i + \mathscr{D}p_i + \mathscr{M}\alpha_i]\frac{\mathrm{d}x}{\ell} \right\} \tag{53}$$

and

$$G_{q_i q_i} = \int_{\text{deck}} q_i^2(x)\frac{\mathrm{d}x}{\ell} \quad [q_i = h_i, p_i \quad \text{or} \quad \alpha_i], \tag{54}$$

and where the buffeting terms $\mathscr{L}$, $\mathscr{D}$, $\mathscr{M}$ will be described later. Note that this form conservatively implies full coherence of flutter derivative action along the span. Alternative choices are easily and routinely incorporated, as desired.

5.2. Flutter

With damping restricted to only the principal elements the single-degree flutter criterion reduces to

$$H_1^* G_{h_i h_i} + P_1^* G_{p_i p_i} + A_2^* G_{\alpha_i\alpha_i} \geq \frac{4\zeta_i I_i}{\rho B^4 \ell}\left[1 + \frac{\rho B^4 \ell}{2I_i} A_3^* G_{\alpha_i\alpha_i}\right]^{1/2}. \tag{55}$$

5.3. Buffeting

Rewriting equations (52) and (53) yields the form

$$\ddot{\xi}_i + 2\gamma_i\omega_{i0}\dot{\xi}_i + \omega_{i0}^2\xi_i = \frac{\rho U^2 B^2 \ell}{2I_i}\int_{\text{deck}} [\mathscr{L}h_i + \mathscr{D}p_i + \mathscr{M}\alpha_i]\frac{\mathrm{d}x}{\ell}, \tag{56}$$

where damping ζ_i and frequency ω_i have been replaced by γ_i, ω_{i0}, their respective aerodynamically influenced counterparts:

$$\omega_{i0}^2 = \omega_i^2 - \frac{\rho B^4 \ell}{2I_i}\omega^2 A_3^* G_{\alpha_i\alpha_i} \tag{57}$$

and

$$2\gamma_i\omega_{i0} = 2\zeta_i\omega_i - \frac{\rho B^4 \ell}{2I_i}\omega[H_1^* G_{h_i h_i} + P_1^* G_{p_i p_i} + A_2^* G_{\alpha_i\alpha_i}]. \tag{58}$$

The Fourier transform of equation (56) is

$$[\omega_{i0}^2 - \omega^2 + 2\mathrm{i}\gamma_i\omega_{i0}\omega]\bar{\xi}_i = \frac{\rho U^2 B^2 \ell}{2I_i}\int_{\text{deck}} [\bar{\mathscr{L}}h_i + \bar{\mathscr{D}}p_i + \bar{\mathscr{M}}\alpha_i]\frac{\mathrm{d}x}{\ell}. \tag{59}$$

Multiplying equation (59) by its complex conjugate yields

$$[(\omega_{i0}^2 - \omega^2)^2 + (2\gamma_i\omega_{i0}\omega)^2]\,\bar{\xi}_i\bar{\xi}_i^* = \left[\frac{\rho U^2 B^2 \ell}{2I_i}\right]^2 \iint_{\text{deck}} \Pi(x_A, x_B, \omega)\,\frac{\mathrm{d}x_A}{\ell}\frac{\mathrm{d}x_B}{\ell} \tag{60}$$

where $\overline{(\ \ )}^* =$ complex conjugate of $\overline{(\ \ )}$ and

$$\Pi(x_A, x_B, \omega) = [\bar{\mathscr{L}}(x_A, \omega)\,h_i(x_A) + \bar{\mathscr{D}}(x_A, \omega)\,p_i(x_A) + \bar{\mathscr{M}}(x_A, \omega)\alpha_i(x_A)] \times [\bar{\mathscr{L}}^*(x_B, \omega)\,h_i(x_B) + \bar{\mathscr{D}}^*(x_B, \omega)\,p_i(x_B) + \bar{\mathscr{M}}^*(x_B, \omega)\,\alpha_i(x_B)]. \tag{61}$$

It is at this point that the newly introduced concept of admittance enters the analysis. The lift ($\mathscr{L}$), drag ($\mathscr{D}$) and moment ($\mathscr{M}$) factors above depend on the horizontal and vertical (u, w) components of gusting and are written in their new forms [see equations (22)–(24)]:

$$\mathscr{L} = -K[H_5^* - \mathrm{i}H_6^*]\frac{u}{U} - K[H_1^* - \mathrm{i}H_4^*]\frac{w}{U}, \tag{62}$$

$$\mathscr{D} = -K[P_1^* - \mathrm{i}P_4^*]\frac{u}{U} - K[P_5^* - \mathrm{i}P_6^*]\frac{w}{U}, \tag{63}$$

$$\mathscr{M} = -K[A_5^* - \mathrm{i}A_6^*]\frac{u}{U} - K[A_1^* - \mathrm{i}A_4^*]\frac{w}{U}. \tag{64}$$

The details of gust analysis proceed from this point. What has been stated above constitutes an abbreviated overview of how the newly defined aerodynamic admittances enter the buffeting problem. The details will not be pursued in the present paper.

6. SUMMARY AND CONCLUSION

The present paper first derives and emphasizes the natural analytical relationships between corrective aerodynamic admittances associated with gust forces and certain well-known flutter derivatives. The roles and implications of these relationships are then suggested by brief remarks on the theoretical steps of flutter and buffeting analyses. Particularly, the proper role of aerodynamic admittances as well-defined frequency-dependent functions intrinsic to the analysis, rather than arbitrarily imposed external correction factors, is emphasized. The paper opens new avenues of investigation in which the efficacy of the proposed rationale for aerodynamic admittance can be more closely examined.

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APPENDIX: NOMENCLATURE

A_i^*	twist flutter derivative
B	deck width
D	sectional drag force
$\mathscr{D}$	dimensionless drag factor, buffet
$G_{q_i q_i}$	modal integral ($q_i = h_i,\ p_i$ *or* α_i)
H_i^*	vertical motion flutter derivative
h	sectional vertical displacement
$h_i(x)$	deck horizontal component, mode i
I_i	generalized inertia of entire bridge
K	$B\omega/U$
L	sectional lift force
$\mathscr{L}$	dimensionless lift factor, buffet
ℓ	bridge span
M	sectional moment
$\mathscr{M}$	dimensionless moment factor, buffet
n	frequency in Hz
P_i^*	sway flutter derivative
p	sectional lateral displacement
$p_i(x)$	deck sway component, mode i
Q_i	generalized modal force, mode i
s	dimensionless time or distance, Ut/B
S_{uu}	power spectral density of horizontal (u) gust component
S_{ww}	power spectral density of vertical (w) gust component
S_{uw}	cross-power spectral density of u, w gust components
t	time
U	cross wind velocity
u	horizontal gust velocity
w	vertical gust velocity
x	spanwise coordinate along deck
α	sectional rotation
$\alpha_i(x)$	deck twist component, mode i
γ_i	aerodynamics-influenced total damping ratio, mode i
ζ_i	mechanical damping ratio of mode i

θ	general wind angle of attack
ξ_i	generalized coordinate of mode i
Π	gust cross-coherence integrand
ρ	air density
τ	integration variable
Φ	indicial force-growth function
Φ'	$d\Phi/ds$
χ^2	aerodynamic admittance factor
ω	circular frequency of oscillation (flutter)
ω_{i0}	circular frequency of oscillation (buffet)
ω_i	natural circular frequency of mode i